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Network topology of an experimental futures exchange

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Abstract. Many systems of different nature exhibit scale free behaviors. Economic systems with power law distribution in the wealth are one of the examples. To better understand the working behind the complexity, we undertook an experiment recording the interactions between market participants. A Web server was setup to administer the exchange of futures contracts whose liquidation prices were coupled to event outcomes. After free registration, participants started trading to compete for the money prizes upon maturity of the futures contracts at the end of the experiment. The evolving 'cash' flow network was reconstructed from the transactions between players. We show that the network topology is hierarchical, disassortative and small-world with a power law exponent of 1.02 ± 0.09 in the degree distribution after an exponential decay correction. The small-world property emerged early in the experiment while the number of participants was still small. We also show power law-like distributions of the net incomes and inter-transaction time intervals. Big winners and losers are associated with high degree, high betweenness centrality, low clustering coefficient and low degree-correlation. We identify communities in the network as groups of the like-minded. The distribution of the community sizes is shown to be power-law distributed with an exponent of 1.19 ± 0.16.

PACS. 89.65.Gh Economics; econophysics, financial markets, business and management – 89.75.Da Systems obeying scaling laws

1 Introduction

Many complex systems exhibit distributions of observables that are not characterized by a single scale. Examples include net wealth, earthquake magnitudes and gene expression [1]. Heterogeneity in system constituents and/or in the interactions among them might underlie the complexity. Continuing advances in information technology have facilitated acquisition and analysis of sheer amounts of data, unraveling the interacting networks of different kinds ranging from the transportation network of airlines in technology [2], collaboration networks of scientists in sociology [3] and binding networks of proteins in biology [4]. Network topologies evolve to fulfill system requirements. Studies of networked systems thus help better understand complex systems. Among the encouraging examples are the jamlessness of scale-free communication networks [5], short separation of small-world acquaintance networks [6] and robustness against random mutations of scale-free biological networks [7]. Further applications of network analysis involve demarcation between social and nonsocial networks by an attribute that measures the correlation between the degrees of interacting nodes [8]. The finding of hierarchical structures in metabolic networks also has implications for functional categorization of metabolites [9].

Financial markets, consisting of such heterogeneous agents as investors, hedgers and arbitragers, show stylized distributions of returns and wealth [10,11]. Intrigued by the universal behavior, physicists have applied the methodologies of nonequilibrium statistical mechanics to elucidating the mechanisms underlying the complexity [12]. Examples include critical phenomenon [13] and self-organized criticality [14] modeling of economic systems.

In line with the network approach to technological, social and biological complex systems, we designed an experimental market, recording every transaction between pairs of participants during the experiment. Transactions (edges) hold information on the flow of assets from sellers to buyers (nodes). Characterization of the evolving topology of the resulting network helps shed light on the emergence of complexity in financial markets. The unique feature of our experiment is that no parallel can be easily undertaken in real markets. We describe the experimental

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settings and market rules in Section 2, followed by characterization of the network by mean shortest path lengths and degree distributions in Section 3. Further analysis in Section 4 unravels subtle network structures including hierarchy, dissortativity and community. We argue that an integrated model of financial markets should accommodate the results of our empirical study.

2 Experiment

A 24 h exchange market was established on the Web, accepting bid and ask orders from registered players via the Internet [15–17]. Upon registration, which was anonymous and free, an account with 30 000 units of fictitious money was allocated to the player on the exchange server. The futures contracts that our market issued were tied to the candidates running for the Taipei mayoral election which took place on December 9, 2006[1]. The liquidation price of each futures contract was determined by the percentage of votes the candidate received on the election day. Such an experiment was run continuously for 30 days, ending on the election day. After the experiment, any contracts in the players' accounts were liquidated using the official counts released by the government. Futures markets in which contract payoffs depend on election outcomes were pioneered by the Iowa Electronic Markets using real money [18]. As incentives in our experiment, money prizes were awarded to the top ten winners determined by the accumulated wealth in the players' accounts. In a previous publication [17], we demonstrated that such a market, which drew typically 400 participants, exhibited power-law-like distributions of price changes, net wealth and inter-transaction times that are characteristic of real world markets. Furthermore, predictions of the market have so far been consistent with election outcomes. In this paper, we examine the evolving network of 'cash' flow recorded along the experiment.

Five candidates ran for Taipei mayor. We included a sixth futures contract to account for invalid ballots. The sum of the six prices should be 100 if the players were rational or the market was efficient. Figure 1 plots this price and volume time series covering the experiment. To convert from irregular to regular temporal spacing time-series in Figure 1, we averaged (summed) the transaction prices (trading volume) within a window of one minute. In the case of missing price data in a time window, the averaged price from the previous window is used. The averaging was to reduce the pricing errors since our market imposed no constraints on the range of price changes between successive transactions. The one-minute window size was chosen as a compromise between error reduction and temporal resolution. The intermittence of price spikes may be attributable to a multiplicative process with additive noise which is known to yield power law distributions in the fluctuations [19,20]. From the number of time points

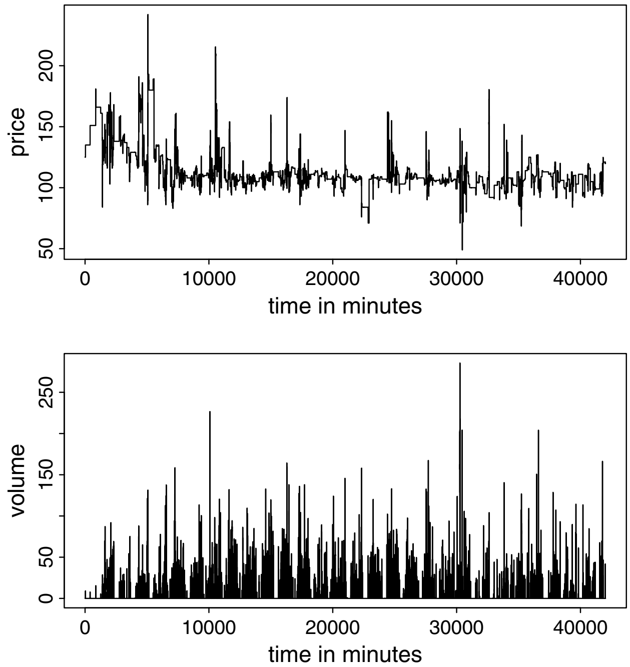

Fig. 1. Price and volume time-series. The time resolution is one minute. Prices (volumes) from multiple transactions within one minute are averaged (summed).

where the trading volumes are nonzero, we determine that the market was active 12.7% of the time. We advertised the experiment by constant posts to the electronic bulletin boards of the colleges throughout Taiwan during the experiment. The number of registrants increased roughly monotonically with time, topping at 628 in the end of the experiment. Trading orders submitted by players were stored in the orderbooks on the server with the continuous double auction mechanism for order matching and price finding. The number of successful transactions in the experiment totaled 8 563. Information on each of the transactions, including price, volume, contract, buyer, seller and time, was recorded.

3 Results and analysis

3.1 Small world cash flow networks

When the bid order of player i was matched with the ask order of player j at a price p and specified volume v of a futures contract, an amount of cash $p \times v$ flew from player i to j. Every day, the server output the cumulative cash flow between any pair of players, from which we reconstructed 30 networks of cash flow, one for each day. On average, $23 \pm 2\%$ of the nodes in the networks were isolated, corresponding to those who registered but had never traded with others. Figure 2 shows the network on day 3. The day 1 network consists only of three isolated nodes. The day 2 network has 11 nodes with 8 connected as shown in Figure 10. We exclude the isolated nodes in the following analysis. The average number of (undirected) edges per

[1] Contract specifications and trading rules were announced at `http://socioecono.phys.sinica.edu.tw/exchange/announce`

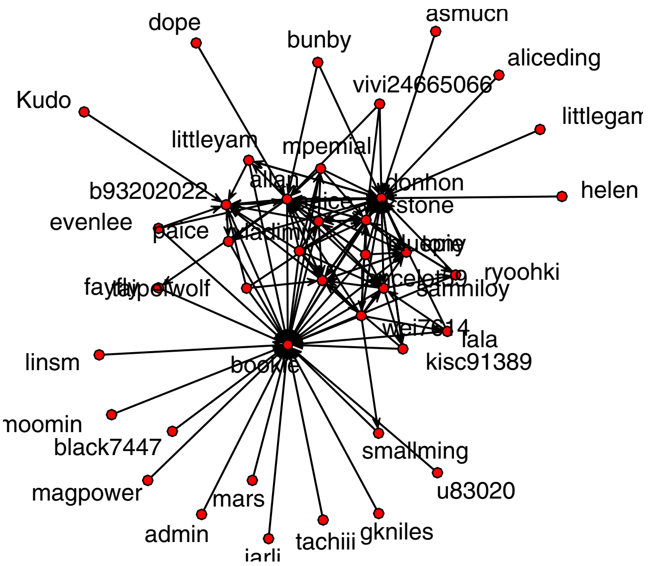

Fig. 2. The trading network on day 3 consisting of 40 interconnected nodes. 15 isolated nodes are not shown.

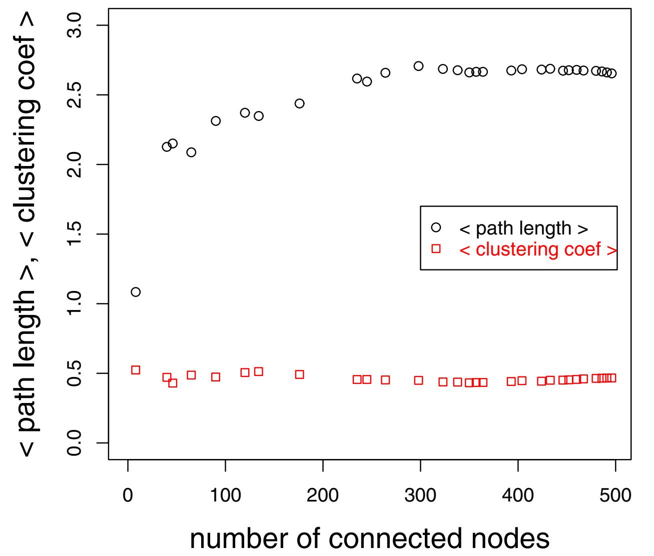

Fig. 3. (Color online) Characteristic path length and clustering coefficient as a function of network size.

node $\langle k \rangle$ in the network increased with day to about 6 within the first 10 days and saturated at around 8 in the final days. Inspection of the networks such as Figure 2 by eyes identifies hubs through which nodes are linked to one another with only few edges. The short separation between two nodes manifests one of the small world properties. To confirm the property, we calculate the characteristic path length of a network which is the average of the smallest numbers of edges between pairs of nodes. The short characteristic path lengths in Figure 3, together with the high clustering coefficients (also shown in Fig. 3 but to be elaborated later), demonstrates the small-worldness of the cash flow networks. As a transaction mediates an exchange of opinions, a small-world market indicates that opinions are efficiently aggregated. In particular, the emergence of the small world properties at early onset of the experiment suggests a low quorum for such a market to be efficient in terms of opinion exchange.

3.2 Degree distribution

The degree distribution $p(k)$ of a network gives the probability of a randomly chosen node to have k edges. A power-law decay of $p(k)$ with k indicates excessive presence of hubs in the network in comparison to random networks. To get the density distribution, we employ bin sizes that are even in the logarithmic scale, with a binsize ratio of 2. Figure 4 shows the distributions of in-degrees, out-degrees and undirected degrees of the cash flow network in the end of the experiment. We found that the degree distributions are well described by a power-law with exponential cut-off,

$$p(k) \sim k^{-\gamma} e^{-k/k_c}. \quad (1)$$

We fitted the data points in Figure 4 to $\log(p) = a - b\log(k) - ck$ by least squares, obtaining $\gamma = b$ and $k_c = 1/c$. The exponents for the in-, out- and undirected degree are found to be $\gamma_{in} = 1.10 \pm 0.15$, $\gamma_{out} = 0.66 \pm 0.13$ and $\gamma = 1.02 \pm 0.09$, respectively. The power-law-like distributions in Figure 4 suggest that the cash flow network from our experiment appears scale free. Note however that the small values of the exponents are in contrast to those of other real world networks found typically in the range $2 < \gamma < 3$ [21]. Hubs play a pivotal role in opinion/information collection and dissemination. If consensus is to be reached independent of the network size, we would expect a wide range of node degrees (corresponding to small γ's) [22]. As we found that, despite transient spikes, the prices of individual contracts were stationary as new players joined throughout the experiment[2], this property may explain the small exponents. In future runs, we plan to test the robustness of the exponent by altering market rules by, e.g., limiting order cancellation or charging transaction fees. Our experiment ended as the futures contracts matured on the election day. The exponential cut-off could be due to such finite size effects as the finite time horizon and low trade activity of the experiment. In the future, we also plan to prolong the tournaments in an effort to studying the origins of the exponential correction.

3.3 Weighted networks and wealth distribution

Flow of cash between players accumulated as time went on. We assign the cumulative flow of cash to the edge. The networks are therefore weighted. The frequency distributions of the weights in Figure 5 show that the weighted cash flows are power-law-like distributed with an exponent of 0.69 ± 0.11. Furthermore, the power-law weights behavior emerged in early stages of the experiment as seen

[2] Time-series plots of the individual contract prices are available at `http://socioecono.phys.sinica.edu.tw/exchange/D/TWMayors06/tw_taipei06-p.jpg`

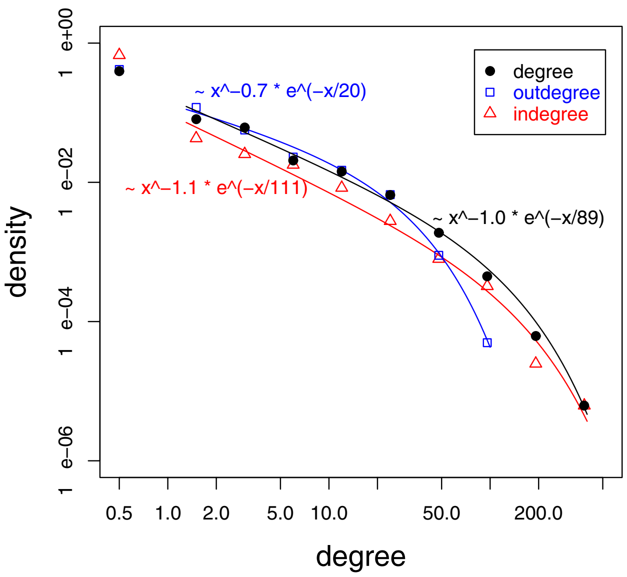

Fig. 4. (Color online) Degree distribution of the cash flow network on the last day. Black dots are from undirected edges. Solid lines are least-squared fits to the data.

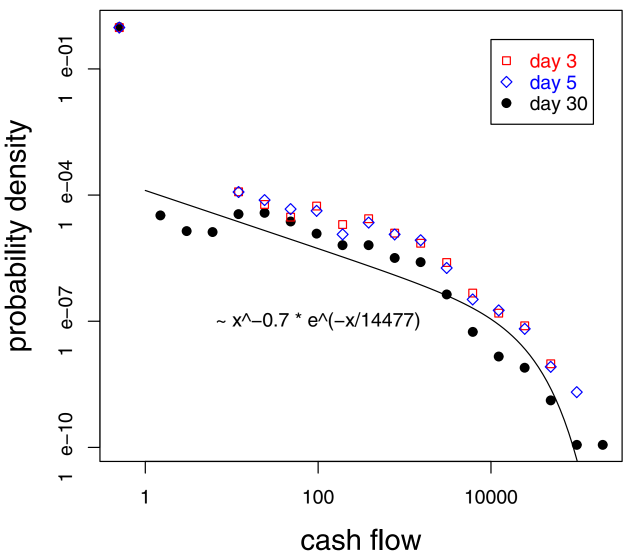

Fig. 5. (Color online) Probability densities of the cumulative cash flow on the edges of the networks on days 3, 5 and 30. Solid line is fit to the day-30 dataset.

from the distributions accumulated up to days 3 and 5 in Figure 5.

We sum the weights on the directed edges pointing to (leaving from) a node to obtain the income (spending) of the node. The incomes (and spending) of the nodes having the same degree are then averaged. A plot of the averaged income versus degree is interesting in that it tells if high in-degree players tend to have high incomes. We found that the income and spending increase with the in- and

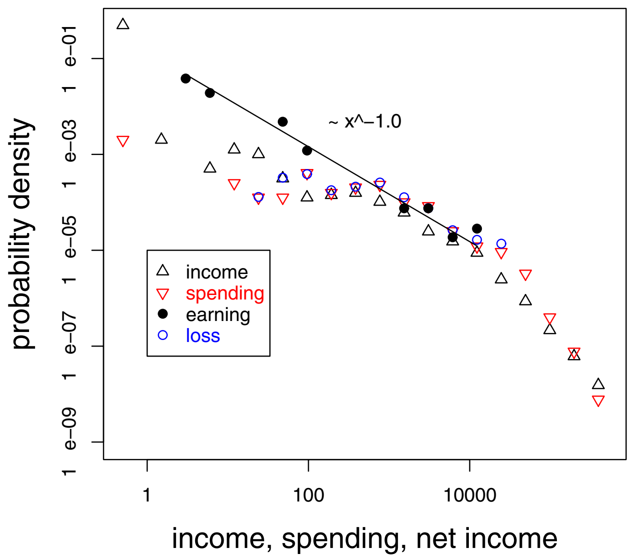

Fig. 6. (Color online) Probability density distributions of the players' incomes, spending and net incomes on the last day. Solid line is fit to the earnings data.

out-degree in a power law fashion,

$$\langle \text{incomes} \rangle \sim k_{in}^{1.26\pm0.05}$$
$$\langle \text{spending} \rangle \sim k_{out}^{1.02\pm0.07}. \quad (2)$$

The relations indicate that those who managed to get more buyers (sellers) cashed in (spent) more. We found no simple functional form relating the in- and out-degrees of the 496 active players up to the last day of the experiment because of the divergence in the scatter plot. However the nonparametric Spearman's rank correlation coefficient between the k_{in} and k_{out} is as high as 0.73. The correlation indicates that those who cashed in more tended to spend more. The same conclusion is reached if we symmetrize the cash flow matrix. That is we replace $w_{ij} = A$ and $w_{ji} = B$ with $w_{ij} = w_{ji} = A + B$ where $w_{ij} = A$ means an amount of A had flown from player i to player j since the experiment began. $A + B$ is then the traded amount between the two players. Using the symmetrized, weighted cash flow matrix, we found a power law similar to equation (2): $\langle \text{traded} \rangle \sim k^{1.15\pm0.04}$ where k is undirected degree.

To get the distribution of net incomes, we subtract the spending from the income of each player. The probability densities in Figure 6 show power law behavior of the incomes, spending, earnings (positive net incomes) and losses (negative net incomes). The exponent of the earnings is 0.99 ± 0.04.

4 Discussion

We have reconstructed the evolving network of 'cash' flow between the players who participated in the trading experiment to compete for real money prizes. We showed

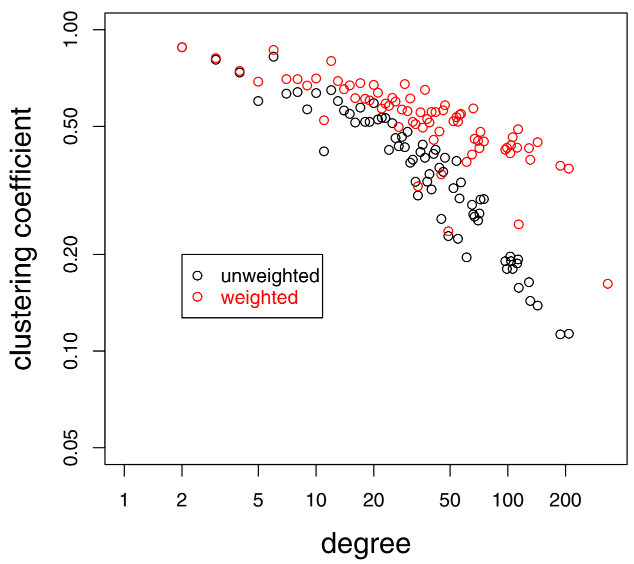

Fig. 7. (Color online) Decrease in the clustering coefficient with degree. The clustering coefficients are calculated from the cash flow network on day 30.

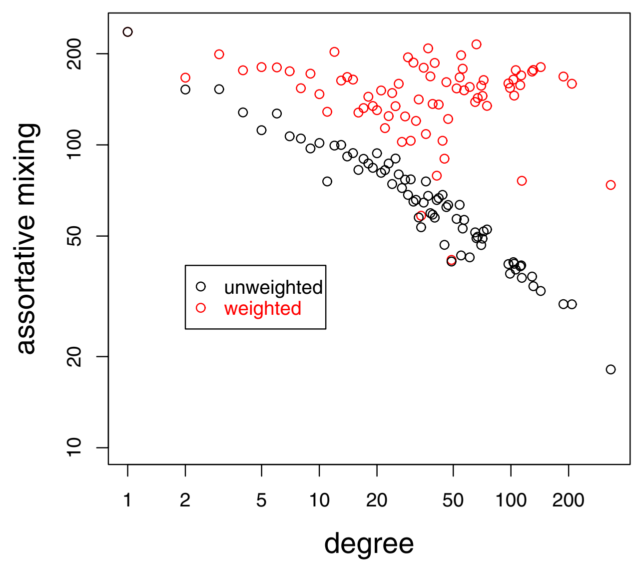

Fig. 8. (Color online) Decrease in assortative mixing with degree. The assortative mixings are calculated from the cash flow network on the last day of the experiment.

that the cash flow network resembled scale free networks with small world properties that emerged very early in the experiment. The distributions of the weights (incomes and spending) also resemble power-law decaying.

4.1 Hierarchical structure

To further characterize the cash flow network, we calculate the clustering coefficient of a player c_i which measures the propensity that her trading partners had traded with one another. We use symmetrized adjacency a_{ij} and cash flow w_{ij} matrices in the calculation of the weighted clustering coefficient [23] which takes into account the frequency or amount of trades between the players,

$$c_i = \frac{1}{s_i(k_i - 1)} \sum_{j,h} \frac{w_{ij} + w_{ih}}{2} a_{ij} a_{ih} a_{jh} \tag{3}$$

where

$$s_i = \sum_j w_{ij} a_{ij}. \tag{4}$$

The clustering coefficients of the players having the same degrees are averaged to get $c(k)$. The results in Figure 7 show a power law decay of the average clustering coefficients $c(k)$ with increasing degree, suggesting a hierarchical architecture in the cash flow network [24]. We also calculated the average clustering coefficient $\langle C \rangle = (1/N) \sum_i c_i$ for each of the 30 networks and found that the values stay rather constant independent of the network size N (cf. Fig. 3), the averages being 0.46 ± 0.02 and 0.52 ± 0.03 for the unweighted (Fig. 3) and weighted versions of the coefficient.

4.2 Disassortative mixing, betweenness and rank

We also calculate the assortative mixing k_{nn_i} which measures the similarity between player i and her trading partners in terms of their degrees [23],

$$k_{nn_i} = \frac{1}{k_i} \sum_j a_{ij} w_{ij} k_j. \tag{5}$$

An analysis parallel to Figure 7 shows a decaying degree-correlation with increasing degree, indicating that the cash flow network is disassortative. The dissortativity may reflect the competitive nature of the market although the dissortativity becomes insignificant considering the weights on the edges (Fig. 8).

Another quantity in network analysis is the betweenness centrality of node i defined as the number of shortest paths between two other nodes passing through i weighted by the inverse of the number of redundancies [25]. We found the mean betweenness centrality $b(k)$ is related to degree by $b(k) \sim k^{2.35\pm0.08}$.

Most of the properties of the node, such as clustering coefficient and betweenness centrality, can be referred to its degree. We rank the players according to their net incomes and plot the degree against rank in Figure 9. The plot shows that high degree players reaped either victory or debacle. The high degree players tend to have low clustering coefficients as in Figure 7. A low clustering coefficient translates that, instead of trading within a clique of partners, the player keeps searching for new investment opportunities across cliques over the network. Whether she wins or loses would then depend on her adaptability to changing opinions.

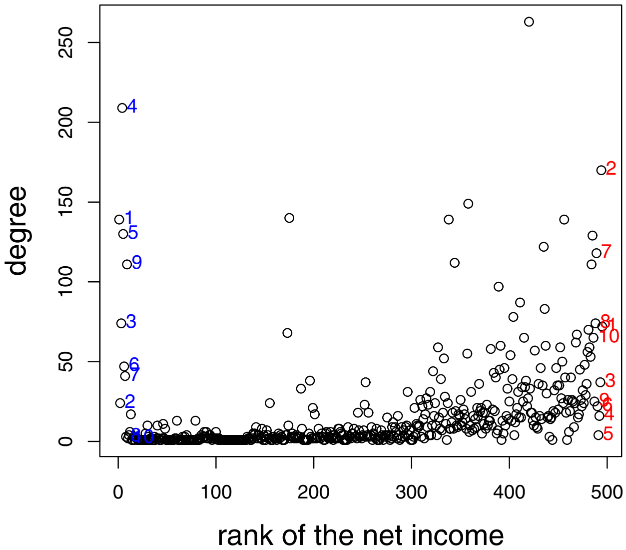

Fig. 9. (Color online) Player's degree versus rank. Top ten winners are marked with blue labels. The bottom 10 players (i.e. losers) are marked with red labels.

4.3 Power-law distribution of community sizes

In the context of our experiment, when the price of a futures contract was considered too high (low), a sell (buy) order was placed. An edge between two nodes in the cash flow network therefore indicates that the two players disagreed to the pricing of the futures contract. In other words, players with no edges linking them were those who thought alike. An algorithm to find communities in the players is thus to partition the cash flow network so that the densities of edges within communities are lower and those between communities are higher than average. An example of such divisions of the network is shown in Figure 10 where it is clear that the within-community edges are minimized while the between-community edges are maximized. We applied the eigenvector-based partitioning algorithm of [26] to the 30 networks and found that the number of communities grew with the number of active players N logarithmically: $-17 + 7\log(N)$. Furthermore, the distribution of community sizes, shown in Figure 11, is found power-law distributed with an exponent of 1.19 ± 0.16. The figure shows three distributions from three cash flow networks on day 19, 25 and 30. The day 19 and 25 networks have, respectively, 20 and 33 communities, corresponding to two extreme deviates (-5 and $+7$) from the logarithmic prediction. Despite of this, the power-law behavior in Figure 11 looks robust. Moreover, the largest communities encompass ~61% of the players.

4.4 Distribution of inter-transaction time intervals

From the volume time-series of Figure 1, we find the time intervals between the points of non-zero trading volumes. The distribution of the inter-transaction times shown in Figure 12 exhibits a truncated power law distribution with exponent 1.28 ± 0.17, consistent with our previous finding [17]. This power law behavior, together with that in

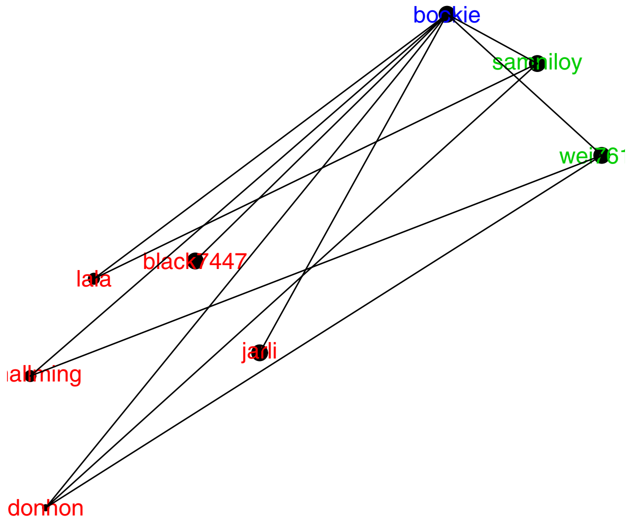

Fig. 10. (Color online) Communities in the cash flow network on day 2. Labels of the same color form communities which, by our partitioning, contain higher-than-average densities of links between communities and lower-than-average densities of links within communities.

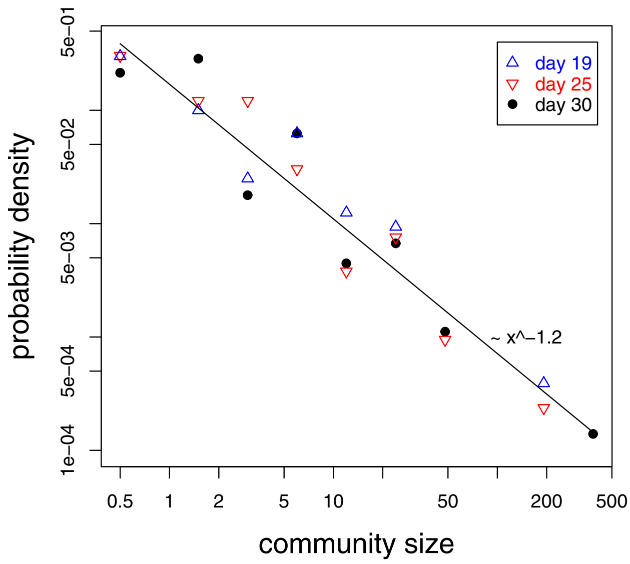

Fig. 11. (Color online) Distributions of the community sizes from the cash flow networks on three terminal days. Solid line is fit to the distribution on day 30, providing the exponent.

the early-day cash flows of Figure 5, may suggest a contribution of human factors [27] to the origins of power laws.

4.5 Effect of bookmaker

Our market includes a built-in bookmaker, referred to as bookie (cf. Fig. 2), which only sells contract portfolios at the fair price of 100 per portfolio [15–17]. Its function is two folds, the first being to allow the few early players, who join right after the experiment starts, to get contracts from the server and initiate selling. The second is for arbitragers to buy portfolios from the server and sell

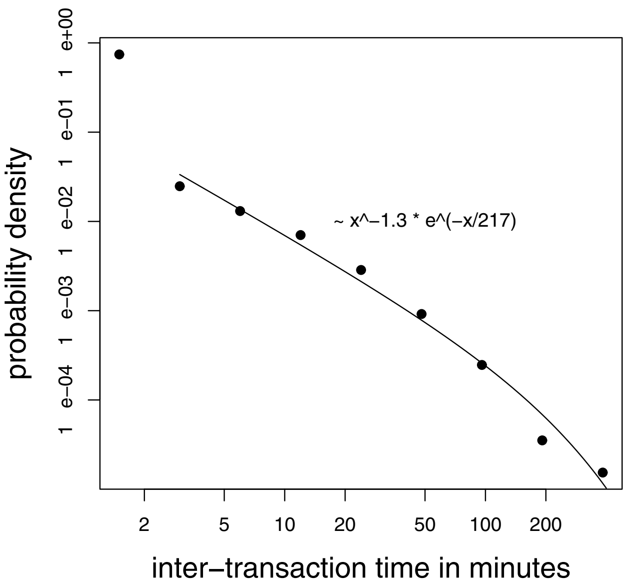

Fig. 12. Power law decay of the time intervals between successive transactions. Solid line is fit to the data.

them to others at a price greater than 100 per portfolio. As seen in Figure 2, bookie is a hub. We examined the effect of bookie by analyzing the bookie-excluded cash flow networks. The conclusions about small-worldness and power-law-likeness remain as there are so many hubs in the networks. However, the weighted clustering coefficients and weighted assortative mixings as a function of degree of the bookie-excluded network converge to the unweighted distributions of Figures 7 and 8 which include bookie. The result indicates again that the cash flow network is hierarchical and disassortative.

In summary, in an effort to study financial markets through network approach, we performed an online experiment in the form of tournament. We recorded the flow of fictitious cash between the 496 registered, active participants throughout the 30-day course of the experiment. The topology of the resulting cash flow networks is found nonrandom with a power-law-like distribution in the connectivity. The heterogeneity in the connectivity as well as weights emerged early in the experiment. The distribution of net incomes in the end of the experiment is also power-law-like distributed. Network analysis indicates that the cash flow network is hierarchical and disassortative. Communities in the network are defined and identified. The distribution of community sizes is power-law distributed, so is the distribution of inter-transaction time intervals. Our experimental platform offers a unique chance of anatomizing such complex systems as financial markets. A better understanding of the complexity calls for models that account for the major findings in the present study.

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